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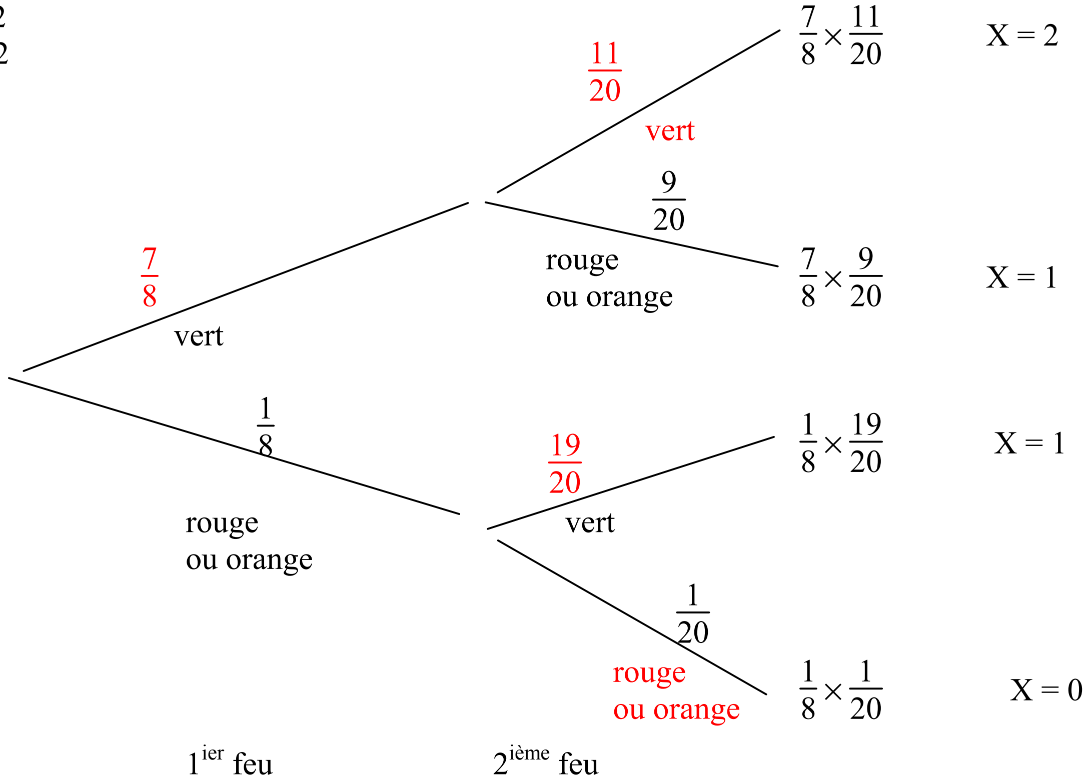
Asie 2002



1° b) $P(X = 0) = \frac{1}{8} \times \frac{1}{20} = \frac{1}{160}$, $P(X = 1) = \frac{7}{8} \times \frac{9}{20} + \frac{1}{8} \times \frac{19}{20} = \frac{82}{120}$, $P(X = 2) = \frac{7}{8} \times \frac{11}{20} = \frac{77}{160}$

c) $E(X) = 0 \times \frac{1}{160} + 1 \times \frac{82}{160} + 2 \times \frac{77}{160} = \frac{82 + 154}{160} = \frac{59}{40} \approx 1{,}475$.

2° a) $P_{E_n}(E_{n+1}) = \frac{1}{20}$ et $P_{\overline{E_n}}(E_{n+1}) = \frac{9}{20}$

b) $E_{n+1} = (E_{n+1} \cap E_n) \cup (E_{n+1} \cap \overline{E_n})$ donc

$p_{n+1} = P(E_{n+1}) = \frac{1}{20} \times P(E_n) + \frac{9}{20} \times P(\overline{E_n}) = \frac{1}{20} p_n + \frac{9}{20}(1 - p_n) = \frac{1}{20} p_n + \frac{9}{20} q_n$

c) $p_{n+1} = \frac{1}{20} p_n + \frac{9}{20}(1 - p_n) = \frac{1}{20} p_n + \frac{9}{20} - \frac{9}{20} p_n = \frac{9}{20} - \frac{8}{20} p_n = \frac{9}{20} - \frac{2}{5} p_n$

3° $U_n = 28\, p_n - 9 \Leftrightarrow 28\, p_n = 9 + U_n$

$U_{n+1} = 28\, p_{n+1} - 9 = 28\left(\frac{9}{20} - \frac{2}{5} p_n\right) - 9 = \frac{28 \times 9}{20} - 28 \times \frac{2}{5} p_n - 9 = \frac{63}{5} - \frac{2}{5} \times (U_n + 9) - \frac{45}{5} = \frac{18}{5} - \frac{2}{5} U_n - \frac{18}{5} = -\frac{2}{5} U_n$

U_n est géométrique de raison $-\frac{2}{5}$.

b) $U_n = \left(-\frac{2}{5}\right)^{n-1} \times U_1 = (28 \times p_1 - 9) \times \left(-\frac{2}{5}\right)^{n-1} = \left(28 \times \frac{1}{8}\right) \times \left(-\frac{2}{5}\right)^{n-1} = \frac{7}{2} \times \left(-\frac{2}{5}\right)^{n-1}$

$p_n = \frac{U_n + 9}{28} = \frac{3{,}5 \times (-0{,}4)^{n-1} + 9}{28} = \frac{1}{8} \times \left(-\frac{2}{5}\right)^{n-1} + \frac{9}{28}$

c) $\left|-\frac{2}{5}\right| < 1$ donc $\lim_{n \to +\infty} U_n = 0$ donc $\lim_{n \to +\infty} p_n = \frac{9}{28}$.